Name ANSWERS | Class

MATHS TEACHER HUB

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Iteration

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

13 The number of animals in a population at the start of year t is P_t
The number of animals at the start of year 1 is 400

Given that

$$P_{t+1} = 1.01P_t$$

work out the number of animals at the start of year 3

Start of year 2 = 1.01 × 400
= 404
start of year 3 = 1.01 × 404
= 408.04

408

(Total for Question 13 is 2 marks)

13 The number of slugs in a garden t days from now is p_t where

$p_0 = 100$

$p_{t+1} = 1.06p_t$

Work out the number of slugs in the garden 3 days from now.

$P_1 = 1.06(100) = 106$

$P_2 = 1.06(106) = 112.36$

$P_3 = 1.06(112.36) = 119.1016$

119

Specimen 2 – Paper 2H

(Total for Question 13 is 3 marks)

14 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$(0)^3 + 4(0) = 0$

$(1)^3 + 4(1) = 5$

The value 1 is between 0 and 5

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$4x = 1 - x^3$

$x = \frac{1 - x^3}{4}$

$x = \frac{1}{4} - \frac{x^3}{4}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^{\ 3}}{4}$ twice,

to find an estimate for the solution of $x^3 + 4x = 1$

$x_0 = 0$

$x_1 = 0.25$ or $\frac{1}{4}$

$x_2 = 0.24609375$ or $\frac{63}{256}$

$\frac{63}{256}$

(3)

(Total for Question 14 is 6 marks)

15 (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

$$(0)^3 + 7(0) - 5 = -5$$
$$(1)^3 + 7(1) - 5 = 3$$

0 is between -5 and 3 so there is a solution

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2 + 7}$

$$x^3 + 7 = 5$$
$$x(x^2 + 7) = 5$$
$$x = \frac{5}{x^2+7}$$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{5}{x_n^{\,2} + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

$$x_1 = \frac{5}{1^2+7} = 0.625$$
$$x_2 = \frac{5}{Ans^2+7} = 0.6785327696$$
$$x_3 = \frac{5}{Ans^2+7} = 0.6704483001$$

0.670

(3)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$, comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

$$(0.670)^3 + 7(0.670) - 5 = -0.009237$$

which is very close to zero, so it is an accurate estimate

(2)

(Total for Question 15 is 9 marks)

16 At the start of year n the population of a species is P_n

At the start of the following year the population of the species is given by

$$P_{n+1} = kP_n \quad \text{where } k \text{ is a positive constant.}$$

The population of the species at the start of year 1 is 8 million. P_1
The population of the species at the start of year 2 is 6 million. P_2

(a) Work out the population of the species at the start of year 3

$P_2 = kP_1$

$6 = k \times 8$

$6 = 8k$

$\frac{6}{8} = k$

$\boxed{k = \frac{3}{4}}$

$P_3 = kP_2$

$= \frac{3}{4} \times 6$

$= 4.5$

4.5 million

(3)

At the start of year 5 the value of k is increased by 0.3 to a new constant value.

Louise thinks that from the start of year 5 the population of the species would increase year on year.

(b) Is Louise correct?
You must give a reason for your answer.

$0.75 + 0.3 = 1.05$

yes because k would be greater than 1

(1)

(Total for Question 16 is 4 marks)

16 (a) Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1, x_2 and x_3
Start with $x_0 = 2$

$x_1 = \sqrt[3]{10-2(2)} = 1.817120593$

$x_2 = \sqrt[3]{10-2(\text{ANS})} = 1.853318496$

$x_3 = \sqrt[3]{10-2(\text{ANS})} = 1.846265953$

$x_1 =$ 1.817120593
$x_2 =$ 1.853318496
$x_3 =$ 1.846265953
(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

(b) Find the value of a and the value of b.

$x = \sqrt[3]{10-2x}$

$x^3 = 10-2x$

$x^3+2x-10 = 0$

$a =$ 2
$b =$ −10
(1)

(Total for Question 16 is 4 marks)

16 Using $x_{n+1} = -2 - \dfrac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$x_1 = -2 - \dfrac{4}{(-2.5)^2} = -2.64$

$x_2 = -2 - \dfrac{4}{(\text{ANS})^2} = -2.573921028$

$x_3 = -2 - \dfrac{4}{(\text{ANS})^2} = -2.603767255$

$x_1 =$ -2.64

$x_2 =$ -2.573921028

$x_3 =$ -2.603767255

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

They are estimates of solutions for the equation $x^3+2x^2+4=0$

$x = -2 - \dfrac{4}{x^2}$ is a rearranged version of $x^3+2x^2+4=0$

(2)

June 2017 – Paper 3H

(Total for Question 16 is 5 marks)

17 (a) Show that the equation $x^3 + 2x - 6 = 0$ has a solution between $x = 1$ and $x = 2$

$(1)^3 + 2(1) - 6 = -3$

$(2)^3 + 2(2) - 6 = 6$

0 is between -3 and 6 so there is a solution.

(2)

(b) Show that the equation $x^3 + 2x - 6 = 0$ can be rearranged to give $x = \dfrac{6}{x^2 + 2}$

$x^3 + 2x = 6$

$x(x^2 + 2) = 6$

$x = \dfrac{6}{x^2 + 2}$

(1)

(c) Starting with $x_0 = 1.45$

use the iteration formula $x_{n+1} = \dfrac{6}{x_n^{\,2} + 2}$ twice to find an estimate

for the solution of $x^3 + 2x - 6 = 0$

Give your answer correct to 4 decimal places.

$x_1 = \dfrac{6}{1.45^2 + 2} = 1.46252228519156$

$x_2 = \dfrac{6}{\text{ANS}^2 + 2} = 1.449634937$

1.4496

(3)

(Total for Question 17 is 6 marks)

17 A ball is thrown upwards and reaches a maximum height.
The ball then falls and bounces repeatedly.

After the nth bounce, the ball reaches a height of h_n
After the next bounce, the ball reaches a height given by $h_{n+1} = 0.55h_n$

After the 1st bounce, the ball reaches a height of 8 metres.

What height does the ball reach after the 4th bounce?

$0.55 \times 8 = 4.4$

$0.55 \times 4.4 = 2.42$

$0.55 \times 2.42 = 1.331$

1.331 metres

(Total for Question 17 is 3 marks)

17 (a) Show that the equation $x^4 - x^2 - 5 = 0$ can be written in the form $x = \sqrt[4]{x^2 + 5}$

$x^4 = x^2 + 5$

$x = \sqrt[4]{x^2+5}$

(1)

(b) Starting with $x_0 = 1.5$

use the iteration formula $x_{n+1} = \sqrt[4]{x_n^2 + 5}$ three times to find an estimate for a solution of $x^4 - x^2 - 5 = 0$

$x_1 = \sqrt[4]{1.5^2 + 5} = 1.640909017$

$x_2 = \sqrt[4]{ANS^2 + 5} = 1.665398002$

$x_3 = \sqrt[4]{ANS^2 + 5} = 1.669763088$

1.669763088

(3)

(Total for Question 17 is 4 marks)

18 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$$(1)^3 + (1) = 2$$
$$(2)^3 + (2) = 10$$

7 is between 2 and 10 so there is a solution

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$$x^3 + x = 7$$
$$x^3 = 7 - x$$
$$x = \sqrt[3]{7 - x}$$

(1)

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$$x_1 = \sqrt[3]{7 - 2} = 1.709975947$$

$$x_2 = \sqrt[3]{7 - \text{ANS}} = 1.742418802$$

$$x_3 = \sqrt[3]{7 - \text{ANS}} = 1.738849506$$

1.738849506

(3)

(Total for Question 18 is 6 marks)

18 At time $t = 0$ hours a tank is full of water.

Water leaks from the tank.
At the end of every hour there is 2% less water in the tank than at the start of the hour.

The volume of water, in litres, in the tank at time t hours is V_t

Given that

$$V_0 = 2000$$
$$V_{t+1} = kV_t$$

100% − 2% = 98%

write down the value of k.

$k =$ 0·98

November 2017 – Paper 2H

(Total for Question 18 is 1 mark)

21 The number of bees in a beehive at the start of year n is P_n.
The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

2016 = 1.05(9500 − 250) = 9712.5

2017 = 1.05(ANS − 250) = 9935.625

2018 = 1.05(ANS − 250) = 10169.90625

10170

Specimen 1 – Paper 2H

(Total for Question 21 is 3 marks)

21 (a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

$$3x^2 - x^3 + 3 = 0$$

$$3x^2 + 3 = x^3$$

$$3 + \frac{3}{x^2} = \frac{x^3}{x^2}$$

$$3 + \frac{3}{x^2} = x$$

(2)

(b) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \quad \text{with } x_0 = 3.2,$$

find the values of x_1, x_2 and x_3

$$x_1 = 3 + \frac{3}{3.2^2} = 3.29296875$$

$$x_2 = 3 + \frac{3}{ANS^2} = 3.276659786$$

$$x_3 = 3 + \frac{3}{ANS^2} = 3.279420685$$

(3)

(c) Explain what the values of x_1, x_2 and x_3 represent.

Represent solutions to the equation $3x^2 - x^3 + 3 = 0$

(1)

(Total for Question 21 is 6 marks)

22 The number of rabbits on a farm at the end of month n is P_n
The number of rabbits at the end of the next month is given by $P_{n+1} = 1.2P_n - 50$

At the end of March there are 200 rabbits on the farm.

(a) Work out how many rabbits there will be on the farm at the end of June.

April $\Rightarrow$ $P = 1.2(200) - 50 = 190$

May $\Rightarrow$ $P = 1.2(190) - 50 = 178$

June $\Rightarrow$ $P = 1.2(178) - 50 = 163.6$

164
(3)

(b) Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time.

The number of rabits will continue to decrease untill there are zero rabits left.
(1)

(Total for Question 22 is 4 marks)